Chapter 2.10

1 (a) $y = OP = \frac{1}{2}gt^2 = 0.05$ m . (b) $v_x = \frac{20.0}{0.10} = 200$ m s^{-1} .

2 The time to fall to the floor is given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 1.3}{10}} = 0.51$ s .
The horizontal distance traveled is therefore $x = v_x t = 2.0 \times 0.51 = 1.02 \approx 1.0$ m .

3 The time to fall to the floor is given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 3}{10}} = 0.775$ s .
The speed can be found by finding the components of the velocity: $v_x = 5.0$ m s^{-1} , $v_y = -gt = -7.75$ m s^{-1} . Then $v = \sqrt{5.0^2 + 7.75^2} = 9.2$ m s^{-1} .
Alternatively we can use conservation of energy to find:
$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{u^2 + 2gh} = \sqrt{5.0^2 + 2\times 10\times 1.3} = 9.2$ m s^{-1} .

4 (a) Use $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 20}{10}} = 2.0$ s . (b) After 1 s, $v_x = 8.0$ m s^{-1} , $v_y = -gt = -10$ m s^{-1} . The speed is thus $v = \sqrt{8.0^2 + 10^2} = 12.8 \approx 13$ m s^{-1} . (c) The angle is $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(-1.25) = -51^\circ$, i.e. 51° below the horizontal. (d) $v_x = 8.0$ m s^{-1} , $v_y = -gt = -20$ m s^{-1} and so the speed is
$v = \sqrt{8.0^2 + 20^2} = 21.5 \approx 22$ m s^{-1} at an angle $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(-2.5) = -68^\circ$.

5 The times to hit the ground are found from
$y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2\times 4.0}{10}} = 0.894$ s and $\sqrt{\frac{2\times 8.0}{10}} = 1.265$ s . The objects are thus separated by $4.0 \times (1.265 - 0.894) = 1.48 \approx 1.5$ m when they land.

6 The horizontal distance traveled by the object falling from 8.0 m is (see previous problem) $x = v_x t = 4.0 \times 1.265 = 5.06$ m . Thus the speed of the other object must be $v_x = \frac{5.06}{0.894} = 5.66 \approx 5.7$ m s^{-1} .

7 The package will fall in a time $\sqrt{\frac{2\times 200}{10}} = 6.325$ s and so the horizontal distance traveled will be $x = v_x t = 50.0 \times 6.325 = 316 \approx 320$ m .

8 The greatest range is obtained when the launch is at 45°. Let the speed of launch be v. Then: $y = vt\sin 45^\circ - \frac{1}{2}gt^2$. $y = 0$ when $0 = vt\sin 45^\circ - \frac{1}{2}gt^2 \Rightarrow t = \frac{2v\sin 45^\circ}{g}$. This is the time the discus is in the air. Then the range is

$$L = vt\cos 45^\circ = v\frac{2v\sin 45^\circ}{g}\cos 45^\circ = \frac{2v^2\sin 45^\circ\cos 45^\circ}{g} = \frac{2v^2\frac{\sqrt{2}}{2}\frac{\sqrt{2}}{2}}{g} = \frac{v^2}{g}$$ hence $v^2 = gL$. The maximum vertical height is found from conservation of energy: $\frac{1}{2}mv^2 = mgh \Rightarrow h = \frac{v^2}{2g} = \frac{gL}{2g} = \frac{L}{2}$.

9 The keys will fall in a time $\sqrt{\frac{2\times 8.0}{10}} = 1.265\text{ s}$ and so the horizontal distance traveled will be $x = v_x t = 6.0\times 1.265 = 7.6 \approx 8\text{ m}$.

10 The components of velocity are: (a) $v_x = v\cos 40^\circ = 20\cos 40^\circ = 15.3\text{ m s}^{-1}$ and so the graph is a horizontal straight line and (b) $v_x = v\cos 40^\circ - gt = (12.9 - 10t)\text{ m s}^{-1}$ so that graph is a straight line with negative slope as shown in answers in textbook. (c) The acceleration is constant so graph is a horizontal straight line.

11 $v_y = v\sin 40^\circ - gt$. At the highest point this component is zero and so $t = \frac{v\sin 40^\circ}{g} = \frac{4.0\times\sin 40^\circ}{10} = 0.257\text{ s}$. Then from $y = vt\sin 40^\circ - \frac{1}{2}gt^2$ we find $y = 4.0\times 0.257\times\sin 40^\circ - \frac{1}{2}\times 10\times 0.257^2 = 0.331 \approx 0.33\text{ m}$.

12 The velocity components after 1.0 s are: $v_x = v\cos 35^\circ = 6.0\cos 35^\circ = 4.915\text{ m s}^{-1}$ and $v_y = v\sin 35^\circ - gt = 6.0\sin 35^\circ - 10\times 1.0 = -6.559\text{ m s}^{-1}$. Thus $v = \sqrt{(-6.559)^2 + 4.915^2} = 8.196 \approx 8.2\text{ m s}^{-1}$. The direction of the velocity vector is at $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(-\frac{6.559}{4.915}) = -53^\circ$.

13 (a) The horizontal displacement is given by $x = 20t\times\cos 50^\circ = 12.6t$ whise graph is a straight line. (b) The vertical displacement is $y = 20t\times\sin 50^\circ - \frac{1}{2}\times 10t^2 = 15.3t - 5t^2$ whose graph is a concave up parabola.

14 (a) The potential energy is given by $E_P = mgy = 40(20t\times\sin 30^\circ - \frac{1}{2}\times 10t^2) = 400t - 200t^2$ whose graph gives the parabola in the answers in the textbook. (b) The kinetic energy is given by

$E_K = E_T - E_P = \frac{1}{2}mu^2 - (400t - 200t^2) = 800 - 400t + 200t^2$ whose graph gives the parabola in the answers in the textbook.

15 In time t the monkey will fall a vertical distance $y = \frac{1}{2}gt^2$ but so will the bullet and hence the bullet will hit the monkey.

16 (a) The ball covers a horizontal distance of 20 m in 2.0 s and so the horizontal velocity component is $\frac{20}{2.0} = 10 \text{ m s}^{-1}$. (b) An arrow directed vertically down. (c) The horizontal distance between the dots will stay the same. The ball will stay in the air for a shorter time: from $y = \frac{1}{2}gt^2$ we get $20 = \frac{1}{2} \times 20t^2 \Rightarrow t = 1.41 \text{ s}$ as opposed to 2.0 s and so the range will be shorter, $x = v_x t = 10 \times 1.41 = 14.1 \approx 14 \text{ m}$. The vertical distance between the dots will be greater, hence the graph in the answers in the textbook.

17 (a) The ball covers a horizontal distance of 60 m in 2.0 s and so the horizontal velocity component is $u_x = \frac{60}{2.0} = 30 \text{ m s}^{-1}$. The ball climbs to a height of 10 m in 1.0 s and so from $y = \frac{u_y + v_y}{2}t$ we have $10 = \frac{u_y + 0}{2} \times 1.0 \Rightarrow u_y = 20 \text{ m s}^{-1}$. (b) The angle of launch is $\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}(\frac{20}{30}) = 34^\circ$. (c) The vertical component of velocity becomes zero at 1.0 s and so $v_y = u_y - gt \Rightarrow 0 = 20 - g \times 1.0 \Rightarrow g = 20 \text{ m s}^{-2}$. (d) The velocity is horizontal to the right and the acceleration is vertically down. (e) With $g = 40 \text{ m s}^{-2}$, the ball will stay in the air for half the time and so will have half the range. The maximum height is reached in 0.50 s and is $y = u_y t - \frac{1}{2}gt^2 = 20 \times 0.50 - \frac{1}{2} \times 40 \times 0.50^2 = 5.0 \text{ m}$ i.e. half as great as before. This leads to the graph in the answers in the textbook.

18* To answer this question we must have formulas for the range and the maximum height. In general we have that: $x = u_x t$ and $y = u_y t - \frac{1}{2}gt^2$. The time to get to the top is found from $0 = u_y - gt \Rightarrow t = \frac{u_y}{g}$ and so the time to cover the range is double this i.e. $\frac{2u_y}{g}$. Then $x = u_x t \Rightarrow 30 = u_x \frac{2u_y}{g}$ and $y = u_y t - \frac{1}{2}gt^2 \Rightarrow 12 = u_y \frac{u_y}{g} - \frac{1}{2}g\left(\frac{u_y}{g}\right)^2$. Simplifying these two equations we get:

$u_x u_y = 150$

$(u_y)^2 = 240$

The second implies that $u_y = 15.5 \text{ m s}^{-1}$ and the first that $u_x = 9.7 \text{ m s}^{-1}$. The speed is thus $v = \sqrt{9.7^2 + 15.5^2} = 18 \text{ m s}^{-1}$ at $\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}(\frac{15.5}{9.7}) = 58^\circ$ to the horizontal.

19 The initial velocity components are: $u_x = 20.0\cos 48^\circ = 13.38 \text{ m s}^{-1}$ and $u_y = 20.0\sin 48^\circ = 14.86 \text{ m s}^{-1}$. The ball hits the sea when the vertical displacement is $y = -60.0 \text{ m}$. Thus $y = u_y t - \frac{1}{2}gt^2 \Rightarrow -60.0 = 14.86t - 5.00t^2$. Solving for the positive root we find $t = 5.26 \text{ s}$. Hence $v_x = u_x = 13.38 \text{ m s}^{-1}$ and $v_y = u_y - gt = 14.86 - 10 \times 5.26 = 37.74 \text{ m s}^{-1}$. The speed at impact is thus $v = \sqrt{13.38^2 + (-37.74)^2} = 40 \text{ m s}^{-1}$ at $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}(-\frac{37.74}{13.38}) = -70^\circ$ to the horizontal. (b) Some of the kinetic energy of the ball will be converted into thermal energy and so the speed at impact will be less. The horizontal component of velocity will decrease in the course of the motion and will tend to go to zero but the vertical component will never become zero (after reaching the maximum height). This means that the angle of impact will be steeper.

20* The components of displacement are given by $x = vt\cos\theta$ and $y = vt\sin\theta - \frac{1}{2}gt^2$. Clearly $x = d\cos\phi$ and so $d\cos\phi = vt\cos\theta \Rightarrow t = \frac{d\cos\phi}{v_0\cos\theta}$. Then $y = v_0\frac{d\cos\phi}{v_0\cos\theta}\sin\theta - \frac{1}{2}g\left(\frac{d\cos\phi}{v_0\cos\theta}\right)^2$. But also $y = d\sin\phi$ and so

$$d\sin\phi = v_0\frac{d\cos\phi}{v_0\cos\theta}\sin\theta - \frac{1}{2}g\left(\frac{d\cos\phi}{v_0\cos\theta}\right)^2$$

$$\sin\phi = \frac{\cos\phi\sin\theta}{\cos\theta} - \frac{1}{2}gd\frac{\cos^2\phi}{v_0^2\cos^2\theta}$$

Solving for d we find

$$\begin{aligned} d &= \frac{2v_0^2\cos\theta}{g\cos^2\phi}(\cos\phi\sin\theta - \sin\phi\cos\theta) \\ &= \frac{2v_0^2\cos\theta}{g\cos^2\phi}\sin(\theta - \phi) \end{aligned}$$

21 The net force on the ball is the component of its weight down the incline i.e. $mg\sin\theta$. The acceleration of the ball is thus $g\sin\theta = 5.0 \text{ m s}^{-2}$. The motion of the ball is thus identical to that of ball projected into the air in a uniform gravitational field of strength 5.0 m s^{-2} and so is the same as projectile motion. In the absence of

resistance forces the path will be parabolic. We then have $u_x = 5.0\cos 25^\circ = 4.53 \text{ m s}^{-1}$ and $u_y = 5.0\sin 25^\circ = 2.11 \text{ m s}^{-1}$. The ball reaches the maximum height on the incline when $0 = 2.11 - 5.0t \Rightarrow t = 0.42 \text{ s}$ and so $y = 2.11 \times 0.42 - \frac{1}{2} \times 5.0 \times (0.42)^2 = 0.45 \text{ m}$.

22* After 1.0 s the velocity components are: $v_x = u_x$ and $v_y = u_y - 10$. Then $\tan 20^\circ = \frac{u_y - 10}{u_x}$. The maximum height is reached at a time given by:

$0 = u_y - 10t \Rightarrow t = \frac{u_y}{10}$ and so the maximum height is given by:

$y = 20 = u_y t - \frac{1}{2} g t^2 = u_y \frac{u_y}{10} - 5(\frac{u_y}{10})^2$. This simplifies to: $20 = \frac{(u_y)^2}{20}$ and so $u_y = 20 \text{ m s}^{-1}$. From $\tan 20^\circ = \frac{u_y - 10}{u_x}$ we then find $u_x = \frac{u_y - 10}{\tan 20^\circ} = 27.47 \approx 27 \text{ m s}^{-1}$. The launch speed is then $\sqrt{27^2 + 20^2} = 34 \text{ m s}^{-1}$. (The angle of launch is $\theta = \tan^{-1} \frac{v_y}{v_x} = \tan^{-1}(\frac{20}{27.47}) = 36^\circ$.)