Chapter 2.7

Q1 The work done is $W = Fd\cos\theta = 24\times 5.0\times\cos 0^{\circ} = 120$ J .

Q2 The work done is $W = Fd\cos\theta = 2.4\times 3.2\times\cos 180^{\circ} = -7.7$ J .

Q3 The work done is $W = Fd\cos\theta = 25\times 15\times\cos 20^{\circ} = 352 \approx 3.5\times 10^{2}$ J .

Q4 (a) The work done by each force is: $W_R = Rd\cos 90^{\circ} = 0$, $W_{mg} = mgd\cos 90^{\circ} = 0$, $W_F = Fd\cos 0^{\circ} = 20.0\times 12.0\times\cos 0^{\circ} = 240$ J and $W_f = fd\cos 0^{\circ} = 14.0\times 12.0\times\cos 180^{\circ} = -168$ J . (b) The net work done is therefore $240-168 = 72.0$ J . (c) The change in kinetic energy is the work done and so this is 72.0 J.

Q5 (a) $W_{up} = Fd\cos 180^{\circ} = -(mg)h = -100\times 10 = -1000$ J . (b) The man must exert a force equal to the weight of the mass. $W_{man} = Fd\cos 0^{\circ} = (mg)h = 100\times 10 = 100$ J . Zero since $W_{down} = Fd\cos 0^{\circ} = (mg)h = 100\times 10 = 1000$ J .

Q6 The change in kinetic energy is $\Delta E_K = \frac{1}{2}\times 2.0(0-5.4^2) = -29.16$ J . This equal the work done by the resistive force i.e. $f\times 4.0\times\cos 180^{\circ} = -29.16 \Rightarrow f = 7.3$ N .

Q7 The work done has gone to increase the elastic potential energy of the spring i.e. $W = \frac{1}{2}\times 200\times(0.05^2-0.03^2) = 0.16$ J .

Q8 The elastic potential energy of the spring will be converted to kinetic energy of the block. Hence $\frac{1}{2}\times 150\times 0.04^2 = \frac{1}{2}\times 1.0v^2 \Rightarrow v = 0.49$ m s^{-1}.

Q9 (a) The minimum energy is required to just get the ball at A. Then, $\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2\times 10\times 4.0} = 8.9$ m s^{-1}. At position B, $\frac{1}{2}\times 1.0\times 8.9^2 = 1.0\times 10\times 2.0 + \frac{1}{2}\times 1.0\times v^2 \Rightarrow v = 6.3$ m s^{-1}. (b) At A: $\frac{1}{2}\times 1.0\times 12.0^2 = \frac{1}{2}\times 1.0\times v^2 + 1.0\times 10\times 4.0 \Rightarrow v = 8.0$ m s^{-1}. At B: $\frac{1}{2}\times 1.0\times 12.0^2 = \frac{1}{2}\times 1.0\times v^2 + 1.0\times 10\times 2.0 \Rightarrow v = 10.2$ m s^{-1}.

Q10 At A: $m \times 10 \times 7.0 = m \times 10 \times 4.0 + \frac{1}{2} \times m \times v^2 \Rightarrow v = 7.75 \text{ m s}^{-1}$. At B: $m \times 10 \times 7.0 = \frac{1}{2} \times m \times v^2 \Rightarrow v = 11.8 \text{ m s}^{-1}$.

Q11 The total energy at A is $E_A = 8.0 \times 10 \times 12 + \frac{1}{2} \times 8.0 \times 6.0^2 = 1104 \text{ J}$. At B it is $E_B = \frac{1}{2} \times 8.0 \times 12^2 = 576 \text{ J}$. The total energy decreased by $1104 - 576 = 528 \text{ J}$ and this represents the work done by the resistive forces. The distance traveled down the plane is 24 m and so $f \times 24 = 528 \Rightarrow f = 22 \text{ N}$.

Q12 In the absence of friction, all the elastic potential energy of the spring will be converted to kinetic energy of the block. Hence $\frac{1}{2} \times 12 \times 0.1^2 = \frac{1}{2} \times 0.020 v^2 \Rightarrow v = 2.45 \text{ m s}^{-1}$. The energy stored in the spring was $\frac{1}{2} \times 12 \times 0.1^2 = 0.060 \text{ J}$. The frictional force will reduce the energy available by the work it does i.e. by $0.05 \times 0.10 = 0.005 \text{ J}$. Hence the kinetic energy of the ball will be 0.055 J and so $\frac{1}{2} \times 0.020 v^2 = 0.055 \Rightarrow v = 2.34 \text{ m s}^{-1}$.

Q13 (a) See graph in answers in textbook. (b) The work done is the area under the graph and this is $2 \times 4 + \frac{(4+2) \times 6}{2} + 4 \times 2 + \frac{(2+6) \times 2}{2} + 4 \times 6 = 66 \text{ J}$. This work goes into increasing the kinetic energy i.e. $\frac{1}{2} \times 2.0 \times v^2 = 66 \Rightarrow v = 8.1 \text{ m s}^{-1}$.

Q14 (a) The potential energy is given by $E_P = mgH - mgs = 12 \times 10 \times 80 - 12 \times 10 s = 9600 - 120 s$. The kinetic energy is $E_K = E_{Total} - E_P = 9600 - (9600 - 120 s) = 120 s$. (b) Since the distance fallen s is given by $s = \frac{1}{2} g t^2$ the answers in (a) become $E_P = 9600 - 600 t^2$ and $E_P = 600 t^2$. These four equations give the graphs in the answers in the textbook. (e) In the presence of a *constant* resistance force, the graph of potential energy against distance will not be affected. The graph of kinetic energy against distance will be a straight line with a smaller slope since the final kinetic energy will be less. The graph of potential energy against time will have the same shape but will reach zero in a longer time. Similarly, the kinetic energy – time graph will reach a smaller maximum value in a longer time.

Q15 The work done is $mgh = 25 \times 10 \times 10 = 2500 \text{ J}$. The power is thus $\frac{2500}{8.2} = 305 \approx 3.0 \times 10^2 \text{ W}$.

Q16 From $P = Fv$, $F \times \frac{100 \times 10^3}{3600} = 90 \times 10^3 \Rightarrow F = 3240 = 3.24 \times 10^3$ N.

Q17 (a) From $P = Fv$, and $F = Mg = 1.2 \times 10^4$ N we find $v = \frac{2.5 \times 10^3}{1.2 \times 10^4} = 2.1 \text{ m s}^{-1}$.
(b) Most likely some of the power produced by the motor gets dissipated in the motor itself and is not used to raise the block.

Q18 The work done is $mgh = 50 \times 10 \times 15 = 7500$ J. The power is thus $\frac{7500}{125} = 60$ W. (b) $e = \frac{60}{80} = 0.75$. (c) The work required is double and the time is therefore also double, 250 s.

Q19 $P \propto v^3$ but $P = Fv$. F is the engine force that equals the air resistance force R hence $v^3 \propto Rv \Rightarrow R \propto v^2$.

Q20 From $P = Fv$, $F \times \frac{240 \times 10^3}{3600} = 250 \times 10^3 \Rightarrow F = 3750 \approx 3.8 \times 10^3$ N.

Q21 (a) Gravitational potential energy to kinetic energy. (b) Gravitational potential energy to thermal energy. (c) Work done by a force is being converted to gravitational potential energy in the absence of friction or gravitational potential energy plus thermal energy in the presence of friction.

Q22 Electrical energy from the motor is converted to potential energy and thermal energy if the elevator is just pulled up. Normally a counterweight is being lowered as the elevator is being raised which means that the net change in gravitational potential energy is zero (assuming that the counterweight is equal in weight to the elevator). In this case all the electrical energy goes into thermal energy.

Q23 (a) The acceleration of the car in the first 2 s is $a = 2.0 \text{ m s}^{-2}$ and so the net force on the car is 2400 N. Hence $F - 500 = 2400 \Rightarrow F = 2900$ N. (b) The average power is $P_{ave} = Fv_{ave} = 2900 \times 2.0 = 5.8$ kW. This can also be obtained from: kinetic energy at end of 2 s interval is $\frac{1}{2} \times 1200 \times 4.0^2 = 9.6$ kJ. Distance traveled is $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2.0 \times 2.0^2 = 4.0$ m so work done against friction is $500 \times 4.0 = 2.0$ kJ. Hence total work done by engine in 2.0 s is $9.6 + 2.0 = 11.6$ kJ and so power developed is $P_{ave} = \frac{11.6}{2.0} = 5.8$ kW. Clearly the first method is preferable. (c) The acceleration is zero so the net force is zero, hence the engine force must be 500 N, equal to the resistance force. (d) Use $P = Fv = 500 \times 4.0 = 2.0$ kW.
(e) The deceleration is $a = 1.0 \text{ m s}^{-2}$ and so $F + 500 = 1200 \Rightarrow F = 700$ N is the braking force. (e) Throughout the motion chemical energy from the fuel is being converted to thermal energy (in the wheels, the road, the engine and the air and,

during braking, in the brake drums). For just the first 2.0 s chemical energy is also being converted to kinetic energy. Some of the chemical energy is also being converted to sound.

Q24 Applying conservation of momentum, $6.0 \times 4.0 = 14 \times v \Rightarrow v = 1.71 \text{ m s}^{-1}$. The change in kinetic energy is thus $\frac{1}{2} \times 14 \times 1.71^2 - \frac{1}{2} \times 6.0 \times 4.0^2 = -27 \text{ J}$.

Q25 Applying conservation of momentum to find the speed of the other mass we get: $0 = -2.0 \times 3.0 + 4.0 \times v \Rightarrow v = 1.5 \text{ m s}^{-1}$. The kinetic energy of both masses is therefore $\frac{1}{2} \times 2.0 \times 3.0^2 + \frac{1}{2} \times 4.0 \times 1.5^2 = 13.5 \approx 14 \text{ J}$. This kinetic energy must have been supplied by the elastic energy stored in the spring.

Q26 The energy stored in the spring was $\frac{1}{2} \times 120 \times 0.15^2 = 1.35 \text{ J}$. The frictional force will reduce the energy available by the work it does i.e. by $1.2 \times 0.15 = 0.18 \text{ J}$. Hence the kinetic energy of the ball will be 1.17 J and so
$\frac{1}{2} \times 0.400 v^2 = 1.17 \Rightarrow v = 2.42 \text{ m s}^{-1}$. (b) In the absence of friction,
$\frac{1}{2} \times 0.400 v^2 = 1.35 \Rightarrow v = 2.60 \text{ m s}^{-1}$ and so $\frac{2.45}{2.60} \times 100\% = 94\%$.

Q27 The two masses will have equal speeds since they are connected by an inextensible string. Initially the total energy is just potential of $4.0 \times 10 \times 5.0 = 200 \text{ J}$. Finally, we have kinetic and as well as potential when the lighter mass is raised:
$2.0 \times 10 \times 5.0 + \frac{1}{2} \times 2.0 v^2 + \frac{1}{2} \times 4.0 v^2 = 200 \Rightarrow v = 5.8 \text{ m s}^{-1}$. Note: you can also do this by finding the acceleration using methods of chapter 2.5 and then use equations of kinematics.

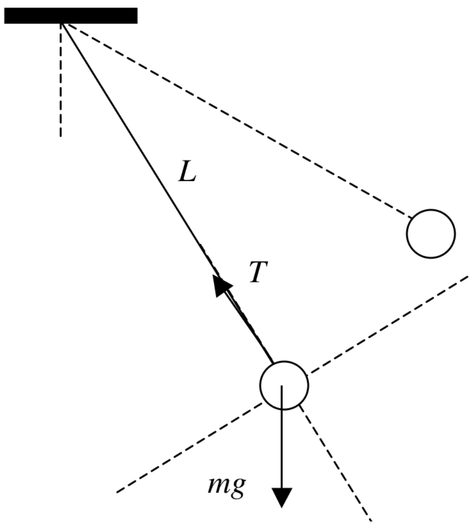
Q28 (a) The acceleration of the mass is $g \sin 30^\circ = 5.0 \text{ m s}^{-2}$ and so the speed is $v = 5.0 \times t$. Hence the kinetic energy (in joule) as a function of time is
$E_K = \frac{1}{2} m v^2 = \frac{1}{2} \times 4.0 \times (5.0 \times t)^2 = 50 t^2$. The distance s traveled down the plane is given by $s = \frac{1}{2} a t^2 = \frac{1}{2} \times 5.0 t^2 = 2.5 t^2$ and so the vertical distance h from the ground is given by $h = 20 - s \sin 30^\circ = 20 - 1.25 t^2$. Hence
the potential energy is $E_P = mgh = 4.0 \times 10 \times (20 - 1.25 t^2) = 800 - 50 t^2$. These are the functions to be graphed with the results as shown in the answers in the textbook.
(b)

Q29 The kinetic energy is $E_K = \frac{1}{2}mv^2$ and the momentum is $p = mv$. Thus $v = \frac{p}{m}$ and so $E_K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$.

Q30 Since no external forces act on the system the two pieces must have equal and opposite momenta (since the original momentum was zero). Thus
$\frac{E_1}{E_2} = \frac{p^2}{2\times\frac{M}{3}} / \frac{p^2}{2\times\frac{2M}{3}} = 2$.

Q31 (a) The net force is zero since the velocity is constant and so $T = mg\sin\theta$. (b) $W_T = Fd = mgd\sin\theta$. (c) $W_W = -mgh = -mgd\sin\theta$. (d) $W_N = 0$ since the angle is a right angle. (e) Zero since the kinetic energy is constant (or zero because $W_W + W_W + W_N = mgd\sin\theta - mgd\sin\theta + 0 = 0$).

Q32 This question requires knowledge of circular motion.



Getting components of the weight we see that the net force along the string direction is $T - mg\cos 20^\circ$ and so $T - mg\cos 20^\circ = \frac{mv^2}{L}$. We need the speed of the ball. Using energy conservation: $\frac{1}{2}mv^2 = mgh = mg(L\cos 50^\circ - L\cos 20^\circ)$, i.e.
$v^2 = 2gL(\cos 50^\circ - \cos 20^\circ)$ and so

$$
\begin{aligned}
T &= mg\cos 20^\circ + \frac{mv^2}{L} \\
&= mg\cos 20^\circ + \frac{m2gL(\cos 50^\circ - \cos 20^\circ)}{L} \\
&= mg\cos 20^\circ + 2mg\cos 50^\circ - 2mg\cos 20^\circ \\
&= mg(2\cos 50^\circ - \cos 20^\circ) \\
&= 6.9 \text{ N}
\end{aligned}
$$

Q33 The graph in the published version of this problem is not a very exact copy of the original. The actual equation of the curve is $v = \frac{1200 \times 30}{1200 + 3.33 \times 30 \times t}$ and this gives the numbers below. (a) In the first 10 s interval the velocity changes from 30 m s^{-1} to 16.4 m s^{-1} and so the average acceleration is $\frac{16.4 - 30}{10} = -1.36 \text{ m s}^{-2}$. In the second 10 s interval the velocity reduces from 16.4 m s^{-1} to 11.3 m s^{-1} and so the average acceleration is $\frac{11.3 - 16.4}{10} = -0.51 \text{ m s}^{-2}$. (b) Because the acceleration is decreasing in magnitude as time goes on. (c) The ratio of the accelerations (and hence forces) is $\frac{1.36}{0.51} \approx 2.7$ and the ratio of the squares of the average speeds is $\left(\frac{21.8}{13.5}\right)^2 \approx 2.6$. The two ratios are approximately the same indicating that the force is proportional to the square of the speed. (d) The distances traveled are $21.8 \times 10 \approx 220 \text{ m}$ and $13.5 \times 10 \approx 140 \text{ m}$. (e) The work doe is equal to the change in the kinetic energy and so $\frac{1}{2} \times 1200(16.4^2 - 30^2) = -3.8 \times 10^5 \text{ J}$ in the first 10 s and $\frac{1}{2} \times 1200(11.3^2 - 16.4^2) = -8.5 \times 10^4 \text{ J}$.

Q34 (a) When the woman reaches the lowest point the rope will be extended by $x = 12.0 \text{ m}$ and she will fallen a distance of $h = 24.0 \text{ m}$. We must have $\frac{1}{2}kx^2 = mgh$ and so $k = \frac{2mgh}{x^2} = \frac{2 \times 60 \times 10 \times 24}{12^2} = 200 \text{ N m}^{-1}$. (b) The man starts with a larger potential energy and so the rope must extend more than 12 m.

Q35 (a) After falling 12.0 m the rope will not have extended and so there will not be any elastic potential energy. Thus $\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh}$ and so $v = 15.5 \text{ m s}^{-1}$. (b) and (c) The speed will keep increasing until the tension in the rope becomes larger than the weight. This happens when $kx = mg \Rightarrow x = \frac{mg}{k} = 3.0 \text{ m}$. When the extension is 3.0 m, conservation of energy gives (we measure heights from this position)

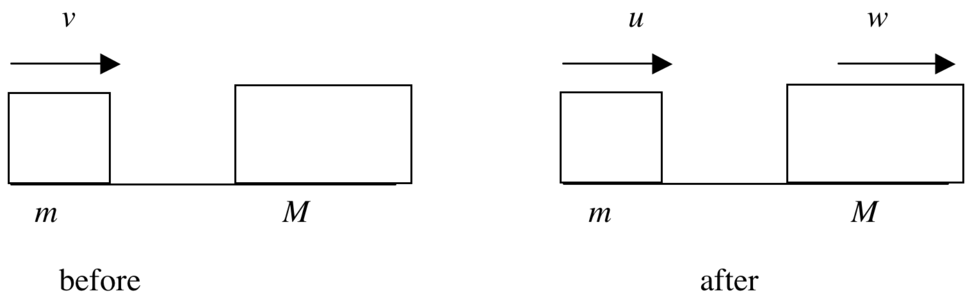
$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = mgh$ where $x = 3.0$ m and $h = 12 + x = 15.0$ m . Then $v = 16.4$ m s^{-1}.
(d) Measure distances from the initial position of the woman on the bridge. For the first 12 m have $\frac{1}{2}mv^2 - mgd = 0 \Rightarrow v = \sqrt{2gd} = \sqrt{20d}$ where d is the distance fallen.
After 12 m the formula for speed changes: $\frac{1}{2}mv^2 + \frac{1}{2}k(d-12)^2 - mgd = 0$ and so $v = \sqrt{20d - \frac{10}{3}(d-12)^2}$. In other words

$$v = \begin{cases} \sqrt{20d} & d \le 12 \\ \sqrt{20d - \frac{10}{3}(d-12)^2} & d > 12 \end{cases}$$

Plotting these on a calculator gives the graph in the answers in the textbook.

Q36 (a) The total kinetic energy before the collision is
$\frac{1}{2} \times 800 \times 5.0^2 + 0 = 1.0 \times 10^4$ J . After the collision it is
$\frac{1}{2} \times 800 \times (-1.0)^2 + \frac{1}{2} \times 1200 \times 4.0^2 = 1.0 \times 10^4$ J . Hence the collision is elastic. (b) The magnitude of the momentum change of either carriage is
$|800 \times (-1.0) - 800 \times 5.0| = 4.8 \times 10^3$ N s and so the average force is
$F = \frac{4.8 \times 10^3}{0.150} = 3.2 \times 10^4$ N . (c) This was done in (b) to give
$|800 \times (-1.0) - 800 \times 5.0| = 4.8 \times 10^3$ N s . (d) The force would have been greater but the impulse would have remained the same. (e) The common speed is 2.0 m s^{-1} and then the total kinetic energy is $\frac{1}{2} \times 800 \times 2.0^2 + \frac{1}{2} \times 1200 \times 4.0^2 = 4.0 \times 10^3$ J . The remaining energy (of 6000 J) is now stored as elastic potential energy in the buffers.

Q37 The diagram is the following. Note that we do not know which way the small mass will move. We assume it is to the right. If the solution gives a negative answer for u then the mass is in fact moving to the left. The equations will determine this.



Applying conservation of momentum: $mv + 0 = mu + Mw$. Applying conservation of kinetic energy: $\frac{1}{2}mv^2 + 0 = \frac{1}{2}mu^2 + \frac{1}{2}Mw^2$. From the first equation, $w = \frac{m(v-u)}{M}$ and substituting in the other gives $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{1}{2}M\left(\frac{m(v-u)}{M}\right)^2$. Expanding out to solve for u gives $0 = m(M+m)u^2 - 2m^2vu + mv^2(m-M)$. This is a quadratic equation for u. Applying the quadratic formula gives

$$u = \frac{2m^2v \pm \sqrt{4m^4v^2 - 4m(M+m)mv^2(m-M)}}{2m(M+m)}$$

$$u = \frac{2m^2v \pm 2mv\sqrt{m^2 - (m^2 - M^2)}}{2m(M+m)}$$

$$u = \frac{2m^2v \pm 2mvM}{2m(M+m)} = \frac{2mv(m \pm M)}{2m(M+m)}$$

The plus sign gives $u = v$ and so $w = 0$ and must be rejected. It is as if one block went through the other. This is not a physical solution for mechanics (but remember this problem when you do waves!). So we take the minus sign to get $u = \frac{v(m-M)}{(m+M)}$.

This means the small block moves to the left. Hence

$$w = \frac{m(v-u)}{M} = \frac{m(v - \frac{v(m-M)}{m+M})}{M} = \frac{2mv}{m+M}$$. The fraction of the kinetic energy transferred to the big mass is therefore:

$$\frac{\frac{1}{2}Mw^2}{\frac{1}{2}mv^2} = \frac{\frac{1}{2}M\left(\frac{2mv}{m+M}\right)^2}{\frac{1}{2}mv^2} = \frac{4mM}{(m+M)^2}$$

Q38 To solve this problem we must apply the procedure of the previous problem to find (using the notation in Q37): $u = \frac{v(m-M)}{(m+M)} = 10\frac{3.0-8.0}{3.0+8.0} = -4.5 \text{ m s}^{-1}$ and

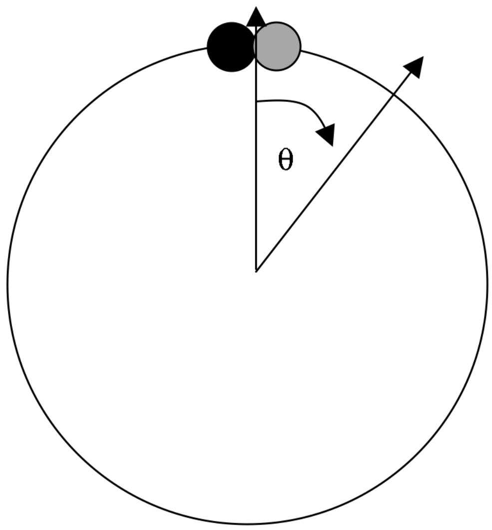
$$w = \frac{2mv}{(m+M)} = \frac{2 \times 3.0 \times 10}{3.0 + 8.0} = 5.5 \text{ m s}^{-1}.$$

Q39 Again we must use the procedure of Q37 to get:

$$u = \frac{v(m-M)}{(m+M)} = 5.0\frac{4.0-6.0}{4.0+6.0} = -1.0 \text{ m s}^{-1}$$ and

$$w = \frac{2mv}{(m+M)} = \frac{2\times 4.0\times 5.0}{4.0+6.0} = 4.0 \text{ m s}^{-1}.$$

Using the diagram below, the first collision takes place at $\theta = 0^\circ$. The masses then move in opposite directions with speeds in the ratio of 4 to 1. Therefore they will collide again at $\theta = \frac{1\times 360^\circ}{5} = 72^\circ$. After the second collision the black ball will stop and the gray will move with a speed of 5.0 m s^{-1}. The situation then starts from scratch but now from an initial position at $\theta = 72^\circ$. So the next collision will be at $\theta = 144^\circ$, i.e. the collisions are taking place at the vertices of a regular pentagon.



Note: You may want to investigate under what conditions collisions will take place at other points on the circle, whether the collisions can take place with the same frequency at *all* points on the circle etc. There is some very nice Physics and Mathematics in this kind of questions.

Q40 (a) Applying conservation of energy we have that the initial kinetic energy gets transformed to potential energy when the mass stops on the wedge and so $\frac{1}{2}mv^2 = mgh$ giving $h = \frac{v^2}{2g}$. (b) If the wedge is free to move, then when the little mass gets to the top it will instantaneously have the same speed as the wedge. Let that speed be *u*. Then by momentum conservation, $mv = (m+M)u$. When at the top the total energy of the system is

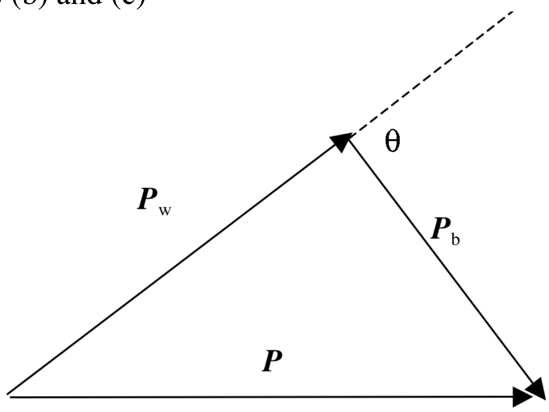
$\frac{1}{2}(m+M)u^2 + mgh = \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} + mgh = \frac{1}{2}\frac{m^2v^2}{(m+M)} + mgh$. This must

equal the original kinetic energy of the small mass and so $\frac{1}{2}\frac{m^2v^2}{(m+M)}+mgh=\frac{1}{2}mv^2$.
Solving for the height we find $gh=\frac{1}{2}v^2-\frac{1}{2}\frac{mv^2}{(m+M)}=\frac{1}{2}\frac{Mv^2}{(m+M)}$ i.e.
$h=\frac{v^2}{2g}\frac{M}{(m+M)}$.

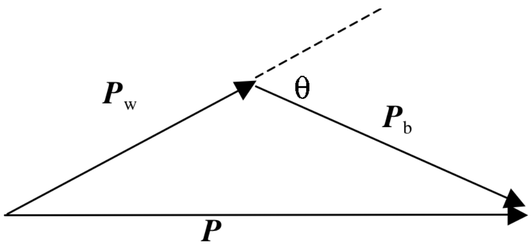
Q41 (a) From $s=\frac{1}{2}at^2$ we find $s=\frac{1}{2}\times 4.0\times 5.0^2=50$ m. (b) At 5.0 s the speed acquired is $v=at=4.0\times 5.0=20$ m s^{-1}. From then on the acceleration becomes a deceleration of $a=g\sin\theta=10\times 0.5=5.0$ m s^{-2}. Then from $v^2=u^2-2as$ we find $0=20^2-2\times 5.0\times s$ giving $s=\frac{20^2}{2\times 5.0}=40$ m. The total distance up the plane is thus 90 m. (c) The car will travel the distance of 90 m from rest with an acceleration of 5.0 m s^{-2} and so from $s=\frac{1}{2}at^2$ we get $90=\frac{1}{2}\times 5.0\times t^2$ giving $t^2=\frac{180}{5.0}=36\Rightarrow t=6.0$ s. (The car took 5.0 s to get to the 50 m up the hill. The remaining 40 m were covered in $0=20-5.0\times t\Rightarrow t=4.0$ s. The time from the start to get back down again is thus 15 s.) (d) For the first 5 s the velocity is given by $v=4.0t$. For the rest of the motion the velocity is $v=20-5.0t$. Graphing these gives the graph in the answers in the textbook. (e) Potential energy: For a distance d traveled up the plane the vertical distance is $h=\frac{d}{2}$ and so the potential energy is $E_P=mgh=0.250\times 10\times\frac{d}{2}=1.25d$. At 90 m, (the highest the car gets on the plane) we have $E_P=1.25\times 90=112.5\approx 112$ J. For the last 90 m (the way down) the graph is decreasing symmetrically. These facts give the graph ion the answers in the textbook. Kinetic energy: For the first 50 m traveled we have that: the speed is given by $v^2=u^2+2as=0+2\times 4.0\times s=8s$ and so the kinetic energy is $E_K=\frac{1}{2}mv^2=\frac{1}{2}\times 0.250\times 8s=s$. (The kinetic energy attained at 50 m is thus 50 J.) In the next 40 m the speed is given by $v^2=u^2+2as=20^2-2\times 5.0\times s=400-10s$ and so the kinetic energy decreases according to $E_K=\frac{1}{2}mv^2=\frac{1}{2}\times 0.250(400-10s)=50-1.25s$. At 40 m the kinetic energy becomes zero. From then on it increases according to $E_K=\frac{1}{2}\times 0.250\times 2\times 5.0\times s=1.25s$. Putting all these together gives the graph in the answers in the textbook. (f) The mechanical energy (kinetic plus potential) will be conserved when there are no external forces acting on the car (other than gravity) i.e. after the first 5.0 s. (g) The motor was exerting a force F up the plane given by

$F - mg\sin\theta = ma \Rightarrow F = ma + mg\sin\theta = 0.250 \times 4.0 + 0.250 \times 10 \times \frac{1}{2} = 2.25\ \text{N}$. The average speed up the plane was $\bar{v} = \frac{0+20}{2} = 10\ \text{m s}^{-1}$ and so the average power is $\bar{P} = F\bar{v} = 2.25 \times 10 = 22.5\ \text{W}$. (g) The maximum power was $P = Fv = 2.25 \times 20 = 45\ \text{W}$.

Q42 (a) (b) and (c)



Momentum must be conserved and so $\vec{P}_w + \vec{P}_b = \vec{P}$. This means the three vectors form a triangle. (d) If the collision is elastic the kinetic energy is conserved. From $\vec{P}_w + \vec{P}_b = \vec{P}$ we deduce taking the dot product $\vec{P}_w + \vec{P}_b = \vec{P} \bullet \vec{P} = (\vec{P}_w + \vec{P}_b) \bullet (\vec{P}_w + \vec{P}_b)$ and since $\vec{P} \bullet \vec{P} = P^2$ and $(\vec{P}_w + \vec{P}_b) \bullet (\vec{P}_w + \vec{P}_b) = P_w^2 + P_b^2 + 2\vec{P}_w \bullet \vec{P}_b$. But $P^2 = P_w^2 + P_b^2$ because of conservation of kinetic energy and hence $\vec{P}_w \bullet \vec{P}_b = 0$ which means that the angle between these two vectors is a right angle. (e) Momentum is still conserved and so the three vectors still make a triangle.



Q43 (a) (i) At A the spring is extended by 0.150 m and so its elastic potential energy is $E_E = \frac{1}{2}kx^2 = \frac{1}{2} \times 4.00 \times 0.150^2 = 0.045\ \text{J}$. Measuring distances from A we have that at B the total energy is $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 - mgh = 0.045\ \text{J}$ where *x* is the extension at B and $h = 0.400\ \text{m}$. The length PB is $\sqrt{0.300^2 + 0.400^2} = 0.500\ \text{m}$ and so the extension is $x = 0.500 - 0.150 = 0.350\ \text{m}$. Putting the numbers in we find for the speed at B: $v=\sqrt{\dfrac{2\times\left(0.045+0.100\times 10\times 0.400-\dfrac{1}{2}\times 4.00\times 0.350^2\right)}{0.100}}=2.00\text{ m s}^{-1}$.

(ii) The tension of the spring is $T=kx=4.00\times 0.350=1.40\text{ N}$. The horizontal component of this force is the force we need. The spring makes an angle with the horizontal whose cosine is $\dfrac{0.300}{0.500}=0.600$ and so the component of the tension is $T_x=1.40\times 0.600=0.840\text{ N}$. (iii) The vertical component of the tension is $T_y=1.40\times\dfrac{0.400}{0.500}=1.12\text{ N}$ and so the net vertical force on the ring is $T_y-mg=1.12-1.00=0.120\text{ N}$ upwards. The acceleration is then upwards and equal to $a=\dfrac{0.120}{0.100}=1.20\text{ m s}^{-2}$. (iv) and (b) The total energy at A is 0.045 J. Measuring distances from A we have that at the lowest point (point L) the total energy is $\dfrac{1}{2}kx^2-mgh=0.045\text{ J}$ where x is the extension at L and h is the distance AL. The length of the spring at L is $\sqrt{0.300^2+h^2}$ and hence the extension of the spring is $x=\sqrt{0.300^2+h^2}-0.150$. Hence $\dfrac{1}{2}\times 4.00\times\left(\sqrt{0.300^2+h^2}-0.150\right)^2-0.100\times 10\times h=0.045$. Using the Solver of the calculator to solve this equation for the height, we find $h=0.698\text{ m}$. (v) After falling a distance h from A we have that the speed of the ring will be given by $\dfrac{1}{2}mv^2+\dfrac{1}{2}kx^2-mgh=0.045\text{ J}$ with $x=\sqrt{0.300^2+h^2}-0.150$. Then $v=\sqrt{0.450+20h-40\left(\sqrt{0.300^2+h^2}-0.150\right)^2}$. Plotting this function on the calculator gives the graph in the answers in the textbook. Asking the calculator to find the maximum of this function and the height at which this occurs gives $v=2.01\text{ m s}^{-1}$ and $h=0.366\text{ m}$ for the height. (c) By conservation of energy the ring will stop at A.