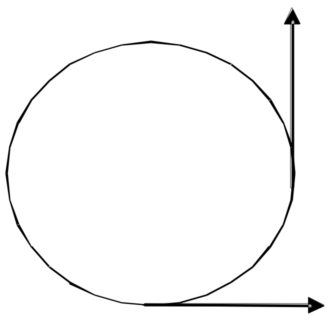
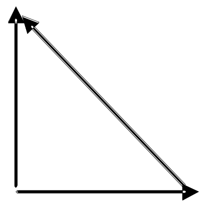
Chapter 2.8

Q1 (a) The average acceleration is defined as $\vec{a} = \dfrac{\Delta\vec{v}}{\Delta t}$. The velocity vectors at A and B and the change in the velocity $\Delta\vec{v}$ are shown below.





The magnitude of the velocity vector is 4.0 m s^{-1} and it takes a time of $\dfrac{2\pi\times 2.0}{4.0} = 3.14 \text{ s}$ to complete a full revolution. Hence a time of $\dfrac{3.14}{4} = 0.785 \text{ s}$ to complete a quarter of revolution from A to B. The magnitude of $\Delta\vec{v}$ is $\sqrt{4.0^2 + 4.0^2} = 5.66 \text{ m s}^{-1}$ and so the magnitude of the average acceleration is $\dfrac{5.66}{0.785} = 7.2 \text{ m s}^{-2}$. This is directed towards north-west and if this vector is made to start at the midpoint of the arc AB it is then directed towards the center of the circle.

(b) The centripetal acceleration has magnitude $\dfrac{v^2}{r} = \dfrac{16.0}{2.0} = 8.0 \text{ m s}^{-2}$ directed towards the center of the circle.

Q2 (a) The centripetal acceleration is $\dfrac{v^2}{r} = \dfrac{4.00}{0.400} = 10.0 \text{ m s}^{-2}$. The tension is the force that provides the centripetal acceleration and so $T = ma = 1.00\times 10.0 = 10.0 \text{ N}$.

(b) From $T = ma = 20.0 \text{ N}$ we have $a = \dfrac{v^2}{r} = 20.0 \text{ m s}^{-2}$ and so $v = \sqrt{20\times 0.40} = 2.83 \text{ m s}^{-1}$. (c) $20.0 = 1.00\times\dfrac{4.00^2}{r} \Rightarrow r = \dfrac{16.0}{20.0} = 0.800 \text{ m}$.

Q3 The centripetal acceleration is $a = \dfrac{v^2}{r} = \dfrac{\left(\dfrac{2\pi r}{T}\right)^2}{r} = \dfrac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$. Hence $f = \sqrt{\dfrac{a}{4\pi^2 r}} = \sqrt{\dfrac{50}{4\pi^2\times 10}} = 0.356 \text{ s}^{-1} \approx 21 \text{ min}^{-1}$.

Q4 The time to complete one revolution is *T*. Thus the number of full revolutions per second is $\frac{1}{T}$. $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$.

Q5 (a) $v = \frac{2\pi R}{T} = \frac{2\pi \times 1.5\times 10^{11}}{365\times 24\times 60\times 60} = 2.99\times 10^4 \approx 30 \text{ km s}^{-1}$.

(b) $a = \frac{v^2}{r} = \frac{(2.99\times 10^4)^2}{1.5\times 10^{11}} = 5.95\times 10^{-3} \text{ m s}^{-2}$.

(c) $F = ma = \frac{mv^2}{r} = 5.98\times 10^{24}\times 5.95\times 10^{-3} = 3.6\times 10^{22} \text{ N}$.

Q6 (a) The angular speed is just $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.24} = 5.1 \text{ rad s}^{-1}$. (b) The frequency is $f = \frac{1}{T} = \frac{1}{1.24} = 0.81 \text{ s}^{-1}$.

Q7 $a = 4\pi^2 r f^2 = 4\pi^2 \times 2.45\times (3.5)^2 = 1.2\times 10^3 \text{ m s}^{-2}$.

Q8 With $a = 9.8 \text{ m s}^{-2}$ we have that

$a = \frac{4\pi^2 r}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 \times 6.4\times 10^6}{9.8}} = 5.08\times 10^3 \text{ s} \approx 85 \text{ min}$.

Q9 (a) Let *v* be the speed on the flat part of the road before the loop is entered. At the top the net force on the cart is its weight and the normal reaction force from the road, both directed vertically downwards. Then, $N + mg = \frac{mu^2}{R} \Rightarrow N = \frac{mu^2}{R} - mg$ where *u* is the speed at the top. For the cart not to fall off the road, we must have $N > 0$ i.e. $u^2 > gR$. From conservation of energy, $\frac{1}{2}mv^2 = mg(2R) + \frac{1}{2}mu^2$ and so $u^2 = v^2 - 4gR$. Hence $v^2 - 4gR > gR$, i.e. $v > \sqrt{5gR}$. (b) For just about equal to $\sqrt{5gR}$ we get $u = \sqrt{gR}$.

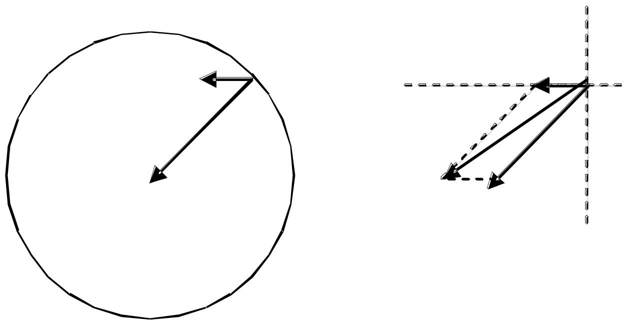
Q10 The speed of rotation around the sun is

$v = \frac{2\pi R}{T} = \frac{2\pi \times 1.5\times 10^{11}}{365\times 24\times 60\times 60} = 2.99\times 10^4 \text{ m s}^{-1}$ and so the (centripetal) acceleration is

$a = \frac{v^2}{r} = \frac{(2.99\times 10^4)^2}{1.5\times 10^{11}} = 5.95\times 10^{-3} \text{ m s}^{-2}$. Hence the force is

$F = ma = \frac{mv^2}{r} = 6.0\times 10^{24}\times 5.95\times 10^{-3} = 3.6\times 10^{22} \text{ N}$.

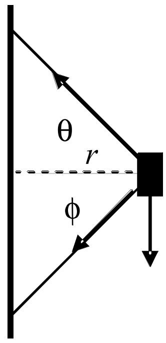
Q11 A point on the surface of the earth covers a circle of radius $R\cos\theta$ where R is the radius of the earth and θ the latitude. Hence the acceleration is $a = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 R\cos\theta}{T^2} = \frac{4\pi^2 \times 6.39\times 10^6 \times \cos 50^\circ}{(24\times 60\times 60)^2} = 0.0217 \text{ m s}^{-2}$. Since $a = \frac{0.0217}{9.8} = 0.0022$, $a = 0.0022g$. The acceleration of a point at the surface of the earth would have two parts. The acceleration due to gravity $g = 9.8 \text{ m s}^{-2}$ directed towards the center of the earth and the centripetal acceleration $a = 0.0217 \text{ m s}^{-2}$ directed as shown (in a diagram that is definitely not to scale).



Taking components, $g_x = -9.8\cos 50^\circ = -6.299 \text{ m s}^{-2}$ and $g_y = 9.8\sin 50^\circ = 7.507 \text{ m s}^{-2}$. Further, $a_x = -0.0217 \text{ m s}^{-2}$ and $a_y = 0$. So the net acceleration has magnitude $\sqrt{7.507^2 + (6.299 + 0.0217)^2} = 9.8 \text{ m s}^{-2}$ (i.e. to the 2 significant figures we are working there is no difference in the acceleration magnitude and $\phi = \tan^{-1}\frac{7.507}{6.299 + 0.0217} = \tan^{-1}\frac{7.507}{6.321} = 49.902^\circ$. The angle off the true vertical is thus $50^\circ - 49.902^\circ = 0.098^\circ$.

Q12 The tension in the string must equal the weight of the hanging mass i.e. $T = Mg$. The tension serves as the centripetal force on the smaller mass and so $T = m\frac{v^2}{r}$. Hence $m\frac{v^2}{r} = Mg \Rightarrow v = \sqrt{\frac{Mgr}{m}}$.

Q13 (a) A diagram is:



Since the lengths of the strings are equal and they make a right angle, $\theta = \phi = 45^\circ$. Further, $r = 1.0 \times \tan 45^\circ = 1.0$ m.

$$T_1 \cos 45^\circ + T_2 \cos 45^\circ = \frac{mv^2}{r} = 4m\pi^2 r f^2$$
$$T_1 \sin 45^\circ = T_2 \sin 45^\circ + mg$$

Putting numbers in, these simplify to

$$T_1 + T_2 = 1117$$
$$T_1 - T_2 = 70.7$$
and so can be solved to give $T_1 = 594$ N, $\quad T_2 = 523$ N.

(b) If the lower string goes slack, $T_2 = 0$. From

$$T_1 \cos 45^\circ = \frac{mv^2}{r}$$
$$T_1 \sin 45^\circ = mg$$

dividing side by side $\tan 45^\circ = \frac{gr}{v^2}$ i.e. $v = \sqrt{gr} = \sqrt{10} = 3.16 \text{ m s}^{-1}$. (c) We now have

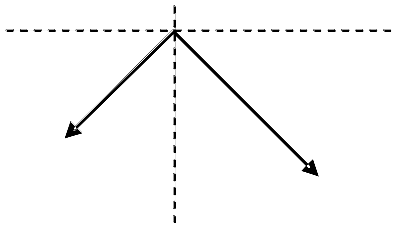
$$T_1 \cos\theta = \frac{mv^2}{r}$$
$$T_1 \sin\theta = mg$$

and again $\tan\theta = \frac{gr}{v^2} = \frac{10r}{1.58^2}$. Now the length of each string is L where $\sqrt{L^2 + L^2} = 2.0$ m hence $L = \sqrt{2.0} = 1.41$ m. Thus, $r = L\cos\theta = 1.41\cos\theta$. Finally,

$\tan\theta = \frac{10 \times 1.41\cos\theta}{1.58^2} = 5.65\cos\theta$. I.e.

$\frac{\sin\theta}{\cos^2\theta} = 5.65 \Rightarrow \frac{\sin\theta}{1 - \sin^2\theta} = 5.65 \Rightarrow 5.65\sin^2\theta + \sin\theta - 5.65 = 0$. This solves to $\sin\theta = 0.915$, i.e. $\theta = 66.3^\circ$. The angle between the string and the vertical is therefore $90^\circ - \theta = 23.7^\circ$.

Q14 At A: $a_t = 3.0 \text{ m s}^{-2}$ and $a_c = \dfrac{v^2}{r} = \dfrac{4.0^2}{4.0} = 4.0 \text{ m s}^{-2}$. The components are as follows:



So $a_x = 4.0\cos 45^\circ - 3.0\cos 45^\circ = \dfrac{\sqrt{2}}{2} \text{ m s}^{-2}$ and

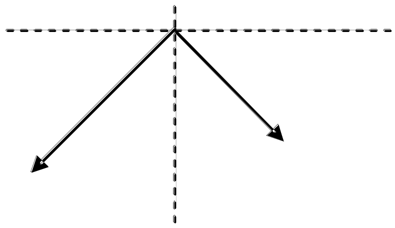
$a_y = -4.0\sin 45^\circ - 3.0\sin 45^\circ = -\dfrac{7\sqrt{2}}{2} \text{ m s}^{-2}$. The net acceleration is thus of

magnitude $a = \sqrt{\left(\dfrac{7\sqrt{2}}{2}\right)^2 + \left(\dfrac{\sqrt{2}}{2}\right)^2} = 5.0 \text{ m s}^{-2}$. (Much easier found from the vector

diagram and Pythagoras' theorem.) The direction is $\theta = \tan^{-1}\dfrac{-\frac{7\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = -82^\circ$ or

278° to the positive x axis.

At B: $a_t = -3.0 \text{ m s}^{-2}$ and $a_c = \dfrac{v^2}{r} = \dfrac{4.0^2}{4.0} = 4.0 \text{ m s}^{-2}$. The components are as follows:



So $a_x = -4.0\cos 45^\circ + 3.0\cos 45^\circ = -\dfrac{\sqrt{2}}{2} \text{ m s}^{-2}$ and

$a_y = -4.0\sin 45^\circ - 3.0\sin 45^\circ = -\dfrac{7\sqrt{2}}{2} \text{ m s}^{-2}$. The net acceleration is thus of

magnitude $a = \sqrt{\left(\dfrac{7\sqrt{2}}{2}\right)^2 + \left(\dfrac{\sqrt{2}}{2}\right)^2} = 5.0 \text{ m s}^{-2}$. (Much easier found from the vector

diagram and Pythagoras' theorem.) The direction is $\theta = 180^\circ + \tan^{-1}\dfrac{-\frac{7\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = 262^\circ$ to

the positive x axis.

Q15(a) By conservation of energy, $mgh = \frac{1}{2}mv^2$ and so

$v = \sqrt{2gh} = \sqrt{2\times10\times120} = 50 \text{ m s}^{-1}$ (with this speed, this amusement park should not have a license to operate!). (b) The forces on a passenger are the weight and the reaction force R both in the vertically down direction. Thus

$R + mg = m\frac{v^2}{r} \Rightarrow R = m\frac{v^2}{r} - mg$. The speed at the top is found from energy conservation as

$mgH = \frac{1}{2}mv^2 + mg(2r) \Rightarrow v^2 = 10\times240 - 2\times10\times60 = 1200$. Hence

$R = 60\times\frac{1200}{30} - 600 = 1800 \text{ N}$. (c) Using $v^2 = u^2 - 2as$ we get $0 = 50^2 - 2a\times40$ and so $a = \frac{50^2}{2\times40} = 30 \text{ m s}^{-2}$ (some passengers will be fainting now, assuming they are still alive!).

Q16 (a) By conservation of energy, $mgL = \frac{1}{2}mv^2$ and so

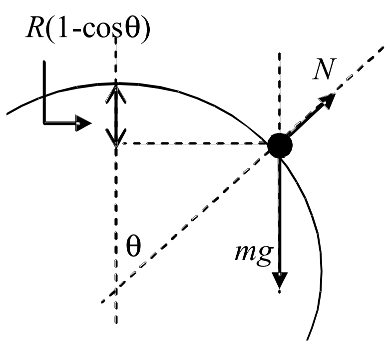
$v = \sqrt{2gL} = \sqrt{2\times10\times2.0} = 6.3 \text{ m s}^{-1}$. (b) $a = \frac{v^2}{r} = \frac{40}{2.0} = 20 \text{ m s}^{-2}$. (c) Tension upwards and a smaller weight downwards.

(d) $T = mg = ma \Rightarrow T = 50 + 5.0\times20 = 150 \text{ N}$ (which is exactly three times the weight).

Q17 $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2\times50.0\times10^3}{(25.0\times10^{-3})^2} = 3.2\times10^9 \text{ m s}^{-2}$.

Q18 $a = 4\pi^2 rf^2 \Rightarrow f = \sqrt{\frac{a}{4\pi^2 r}} = \sqrt{\frac{4\times10}{4\pi^2\times25}} = 0.201 \text{ s}^{-1} \approx 12 \text{ min}^{-1}$.

Q19 A diagram is:



The components of the weight are $mg\sin\theta$ tangent to the circle and $mg\cos\theta$ radially towards the center. The net force in the radial direction is therefore $mg\cos\theta - N$ where N is the normal reaction force. The ball has fallen a distance $R - R\cos\theta$ and so by energy conservation, $mg(R - R\cos\theta) = \frac{1}{2}mv^2$, i.e. the speed is $v = \sqrt{2gR(1-\cos\theta)}$. Hence

$$mg\cos\theta - N = m\frac{v^2}{R}$$

$$N = mg\cos\theta - m\frac{v^2}{R}$$

$$N = mg\cos\theta - m\frac{2gR(1-\cos\theta)}{R}$$

$$N = 3mg\cos\theta - 2mg$$

When $N \to 0$ the ball will fall off the big sphere, i.e. when $\cos\theta = \frac{2}{3}$, independently of the radius of the sphere. This angle is $\theta = \cos^{-1}\frac{2}{3} = 48.2^\circ$.