Chapter 4.6

Q1 A standing (or stationary) wave is a special wave formed when two identical traveling waves moving in opposite directions meet and then superpose. This wave, unlike a traveling wave, has nodes i.e. points where the displacement is always zero. The antinodes, points where the displacement is the largest do not appear to be moving. A standing wave differs from a traveling wave in that it does not transfer energy and that the amplitude is variable.

Q2 A standing (or stationary) wave is a special wave formed when two identical traveling waves moving in opposite directions meet and then superpose.

Q3 (a) A node is a point in the medium where the displacement is *always* zero. (b) An antinode is a point in the medium where the displacement, at some instant, will assume its maximum value.

Q4 We must disturb the string with a frequency that is equal to the frequency of the second harmonic.

Q5 Resonance is the matching of frequencies between a system and a driving force. In other words when a system of natural frequency f is acted upon by a periodic force whose frequency is also f. Examples include (1) a swing that oscillates with some frequency and is then acted upon by a periodic force that has the same frequency making the amplitude of oscillations large and (2) a radio that is tuned to the frequency of a particular radio station. The radio responds only to that frequency.

Q6 This is probably due to a resonance phenomenon between the natural frequency of the steering column of the car and the forces experienced by the car due to a bumpy road.

Q7 If the tension is doubled the speed will increase by $\sqrt{2}$. The wavelength will stay the same (same string length) and so the frequency will increase by $\sqrt{2}$ to 354 Hz.

Q8 The first harmonic has wavelength $2L$ (L is the length of the string) and the second a wavelength L. The ratio of the *frequencies* is then 1 to 2 since the speed is the same.

Q9 If the tension increases by 20% the speed will increase by approximately 10% (since $v \propto \sqrt{T}$. The frequency will then also increase by 10% i.e. it will become approximately 550 Hz.

Q10 (a) The wavelength of the fundamental is $2L = 1.00$ m. The frequency is then $f = \frac{v}{2L} = 225$ Hz. (b) The sound produced by the vibrations of the string will have the same frequency i.e. 225 Hz and so the wavelength of sound will be $\lambda = \frac{c}{f} = \frac{330}{225} = 1.47$ m.

Q11 See text, Fig. 6.8.

Q12 The wavelength of the sound is $\lambda = \frac{330}{306} = 1.08 \text{ m}$. For the fundamental we need $\lambda = 4L \Rightarrow L = \frac{1.08}{4} = 0.270 \text{ m}$ which is outside our range. For the second harmonic $\lambda = \frac{4L}{3} \Rightarrow L = \frac{3 \times 1.08}{4} = 0.810 \text{ m}$ and fir the third $\lambda = \frac{4L}{5} \Rightarrow L = \frac{5 \times 1.08}{4} = 1.35 \text{ m}$. For the next harmonic we have $\lambda = \frac{4L}{7} \Rightarrow L = \frac{7 \times 1.08}{4} = 1.89 \text{ m}$ which is outside our range. So we need only the lengths 0.810 m and 1.35 m.

Q14 The wavelengths in the open tube are given by $\lambda = \frac{2L}{n}$. The frequencies of two *consecutive* harmonics are then ($f = \frac{c}{\lambda} = \frac{cn}{2L}$) $300 = \frac{cn}{2L}$ and $360 = \frac{c(n+1)}{2L}$. This means that $\frac{cn}{300} = \frac{c(n+1)}{360} \Rightarrow 360n = 300n + 300 \Rightarrow n = 5$. We have the fifth and sixth harmonics. Hence $L = \frac{330 \times 5}{600} = 2.75 \text{ m}$.

Q15 (a) The speed of the wave on this string is $v = \sqrt{\frac{90.0}{\frac{3.0 \times 10^{-3}}{0.50}}} = 122.5 \text{ m s}^{-1}$. The wavelength of the fundamental is $\lambda = 2L = 1.0 \text{ m}$ and so the frequency is 122 Hz. (b) The wavelength of the standing wave is still $\lambda = 2L = 1.0 \text{ m}$ and its frequency is 122 Hz. This is also the frequency of the sound in water. Hence the wavelength of the sound in water is $\lambda = \frac{1500}{122} = 12 \text{ m}$.

Q16 The vibration generator indices water standing waves in the container. At the frequency of 0.75 Hz the water spills out because the amplitude is large because we have resonance. The frequency of the water standing waves must be close to 0.75 Hz when this happens. The wavelength of the standing wave as shown is $2L = 2 \times 0.12 = 0.24 \text{ m}$ and so the speed of the water waves is estimated to be $0.24 \times 0.75 \approx 0.18 \text{ m s}^{-1}$.

Q17 This problem contains a typo. The speed of the water waves was meant to be $v = 15 \text{ cm s}^{-1}$.

(a) You will observe that the ripples that are created on the surface of the liquid are stable which means that standing waves are formed. (b) The wavelength is very small, roughly 0.5 cm which means that the standing waves are in a high harmonic mode.

The frequency is about $f=\frac{15}{0.5}=30\text{ Hz}$. This is much higher frequency than that required to form standing waves in the fundamental mode. In the fundamental mode the wavelength would be about twice the diameter of the cup i.e. 16 cm (we have antinodes at each end). With a speed of $v=15\text{ cm s}^{-1}$, the frequency is only $f=\frac{15}{16}\approx 1\text{ Hz}$. This is the kind of frequency that would shake the liquid if you just held the cup in your hand and just walked with it normally (you shake the liquid every time you take a step, i.e. about once per second). (c) The higher frequency in the case of dragging the cup is caused by the frictional forces between the surface and the cup base. The frictional force is sometimes a static frictional force and in other cases a kinetic frictional force. This makes the cup stop and start abruptly, disrupting the liquid surface. This seems to be independent of the speed with which the cup moves.

Q18 (a) A standing wave is made up of two traveling waves. The speed of energy transfer of the traveling waves is taken to be the speed of the standing wave. (b) From $y=5.0\cos(45\pi t)$ we deduce that the frequency of oscillation of point P and hence also of the wave is $\frac{45\pi}{2\pi}=22.5\text{ Hz}$. The wavelength is then $\lambda=\frac{v}{f}=\frac{180}{22.5}=8.0\text{ m}$. Since the diagram shows a second harmonic this is also the length of the string. (c) The phase difference is π and so $y=5.0\cos(45\pi t+\pi)=-5.0\cos(45\pi t)$.

Q19 The frequency of the sound wave is $f=\frac{330}{1.7}=194\text{ Hz}$ and so $\omega=2\pi f=1219\text{ s}^{-1}$. From our formulae in simple harmonic motion, the maximum speed of the molecule will be $v_{\max}=\omega A=1219\times 4.0\times 10^{-7}=4.88\times 10^{-4}\text{ m s}^{-1}$. So the maximum kinetic energy of the molecule is $E_{\max}=\frac{1}{2}mv_{\max}^2=\frac{1}{2}\times 4.8\times 10^{-26}\times 4.88\times 10^{-4}=5.7\times 10^{-33}\text{ J}$.

Q20 (a) The wavelength is 2/3 of the length i.e. 4.0 m. (b) The frequency is then $\frac{120}{4.0}=30\text{ Hz}$ and this is also the frequency with which point P oscillates in SHM. From the general SHM formula for displacement $y=A\cos(\omega t)=A\cos(2\pi ft)$ we deduce that $y=4.0\cos(60\pi t)$. (c) Point Q is out of phase with P by π and so $y_Q=2.0\cos(60\pi t+\pi)=-2.0\cos(60\pi t)$. Point R is in phase with P and so $y_R=2.0\cos(60\pi t)$. (d) From $t=0$ to $t=\frac{T}{4}$, P and Q move distances equal to their respective amplitudes of oscillation. The period is $\frac{1}{30}=0.333\text{ s}$ and so $v_P=\frac{4.0\times 10^{-3}}{0.333/4}=0.048\text{ m s}^{-1}$ and $v_P=\frac{2.0\times 10^{-3}}{0.333/4}=0.024\text{ m s}^{-1}$. (e) The maximum speeds are $v_P = \omega A = 2\pi f A = 2\pi \times 30 \times 4.0 \times 10^{-3} = 0.75 \text{ m s}^{-1}$ and $v_Q = \omega A = 2\pi f A = 2\pi \times 30 \times 2.0 \times 10^{-3} = 0.38 \text{ m s}^{-1}$.